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FEATURE

Scientific Constructivism: A Remedy for Miseducation in Mathematics

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Abstract: *Mathematics education is in serious trouble. Some call this problem "mathematical miseducation." Unsatisfactory performance in mathematics by students at all levels of education—elementary to tertiary—as evidenced through national and international test scores, teacher frustrations in mathematics teaching, and parent complaints about mathematics teaching and learning are commonplace phenomena. With the premise that improvement of mathematics education must begin with teachers, this paper addresses the issues of teachers' understanding of 1) the nature of mathematics and 2) mathematics teaching. Teachers of mathematics need to be proficient in mathematical content as well as in the use of sound mathematical pedagogy. Preservice training of mathematics teachers should include a good balance of these two components. Since teachers teach the way they were taught, these mathematics courses should involve mathematics cognition, not just rote learning. The challenges in teaching mathematics at all levels are daunting and yet too important to ignore. Scientific constructivism is proposed as a partial solution for both the mathematics education of students and of teachers.*

Five-year-old Natasha enjoys her preschool mathematics, excitedly counting objects, clicking numbers on her toy computer at home, and singing nursery rhymes about numbers. As I observe her enthusiasm for mathematics and other learning activities, I cannot help but wonder, "Will Natasha continue to be mathematically healthy throughout her school life and one day exit school mathematically healthy?" or "Will she soon be joining those confessing the cliché, 'I am not good at mathematics'?" "Will Natasha become another victim of mathematics miseducation?"

The paradox in attitudes towards literacy is that, while individuals do not easily admit their inability to read, most people seem to feel little stigma associated with admitting innumeracy, and may be happy to consider mathematics as something they courageously and painfully endured (Battista, 1999). Such is the effect of mathematics miseducation.

Mathematics is one of the school subjects regarded as challenging by both students and teachers alike, from my own observation through the past years as a student, teacher, and parent. Even a general observation of students and teachers gives the impression that mathematics is one of the most misunderstood subject areas in schools. The evidences for this impression are also seen in low test scores, teacher frustrations, and parent complaints—all related to mathematics teaching and learning. Poor performance in mathematics also frequently alienates students from advanced studies. In recent years, mathematics education has experienced problems at all levels—at the elementary and secondary levels, as well as at the tertiary level, where mathematics teachers are prepared (Garcia, 2010; Melanson, 2010; Pytel, 2009). However, the good news is that more recently, mathematics professionals are making serious efforts to understand this "mysterious" subject—its content and its related pedagogy—and to provide directions for practitioners.

The Current Need in Mathematics Education

Mathematics education is becoming an increasingly frequent discussion topic among educators, as evidenced by the plethora of articles in professional journals in recent years. There seem to be *concerns*, and *conflicts* among mathematics professionals about the present concerns of mathematics education, as well as its future. Some of the voices heard in this dialogue sound like these: "The time has come for mathematical scientists to reconsider their role as educators" (Bass, 1997), "Mathematics courses for pre-service elementary teachers are especially important to move from a focus on algorithms toward conceptual understanding and problem solving" (Brown & Reed, 2005). *Mathematics wars*, as Marshall (2003) calls them, have been fought at the professional level in the name of "rote learning" vs. "thinking" and "The Basics" vs. "Reform Mathematics." However, as Tucker (1996) asserts, "The challenges we face in providing the best possible mathematics instruction for our students at all levels are daunting and too important for divisive infighting" (p. 1468).

In the face of these concerns and conflicts, mathematics educators need to collaboratively arrive at some consensus on what a good curriculum and method of instruction for school mathematics should look like. This paper is an attempt to analyze and synthesize the knowledge base of school mathematics education in order to arrive at a reasonable framework for actual practice. The barriers that stand in the way of practitioners may be the tradition that knowledge is transferable—instead of it being actively built up by the learner—and a belief system that de-emphasizes the dynamics of a perceivable world. However, these barriers are easy to overcome if practitioners pause to understand the nature of learners and of mathematics itself.

The Mathematics Learning Needs of Students

A good starting point in rethinking mathematics education is to consider the mathematics learning needs of students in the 21st century. Developing basic skills continues to be as important as ever, however, genuine understanding and personal sense making must be part of mathematics learning. In the modern context, these skills may be translated as problem solving and reasoning skills which mathematics education can and must provide. An implication of societal shifts could mean, as Marshall (2003, p. 194) states, that "children in school today—and tomorrow—will need more mathematics than their parents did yesterday, and they will need to be taught in a far better way." These children will be dealing with more abstract mathematics in the future within the world of computers and technology. Marshall (2003) admonishes that children should stop learning what machines can do, but rather should learn to do things machines cannot do.

How is mathematics taught in a typical classroom? Battista's (1999) vivid description fits the general trend:

> For most students, school mathematics is an endless sequence of memorizing and forgetting facts and procedures that make little sense to them. Though the same topics are taught and retaught year after year, the students do not learn them. Numerous scientific studies have shown that traditional methods of teaching mathematics not only are ineffective but also seriously stunt the growth of students' mathematical reasoning and problem-solving skills. . . . Yet traditional teaching continues, taking its toll (p. 426)

There must be a shift from "teaching" mathematics to "learning" mathematics. From the traditional *transmission* modality where students passively "absorbed" mathematical concepts, students must learn by *constructing* their own mathematical understanding which must also result in *creating* mathematics. This shift will result in students 1) developing structures that are more complex and powerful than what they currently possess, and 2) becoming autonomous and self-motivated in their learning (Clements & Battista, 1990, para. 5-6).

The students of today are the professionals of tomorrow. Mathematics provides the design and tools for the kind of thinking required in most, if not all, professions, be it industry, technology, education, or business. As the world is becoming flat (Friedman, 2005), there is a leveling of the competencies needed for various professions. For example, how many professions *do not* need technology skills? Is not problem solving becoming a universal competency? Surely it is!

It is time for rethinking the way mathematics has been taught—rather than being for the elite and brilliant, mathematics needs to be accessible to *all*. Such a shift will entail a change in mathematics education, both in content and pedagogy. Therefore, it is imperative that a description of the nature of mathematics as a discipline as well as some of the appropriate instructional strategies be identified, from an analysis of current research literature.

The Nature of Mathematics Content

Mathematics, as a discipline, is an exact science. There are precise solutions for each problem, though often there exists more than one way of arriving at them. However, the *process* of problem solving that happens in mathematics is not as neat as what happens in the mathematics class where it is taught. A simple comparison (Otto & van der Ploeg, as cited in Battista, 2001, p. 24) of mathematics in school and how mathematics knowledge is produced is helpful in identifying the contrast between the two processes (see Table 1).

The implication of the comparison between the so-called “refined” mathematics of the school and the “raw” mathematics of the professional field is obvious. Battista aptly states the implications of this difference as, “To be able to use mathematics to make sense of the world, students must first make sense of mathematics” (2001, p. 23). He asserts that such learning uses scientific constructivism. “We do mathematics when we recognize and describe patterns; construct physical and/or conceptual models of phenomena; create symbol systems to help us represent, manipulate, and reflect on ideas; and invent procedures to solve problems” (1999, p. 428). Students need to experience more of these “raw” mathematical activities in order to appreciate the “refined” mathematics more fully.

Use of Scientific Constructivism in Mathematics

Decisions about teaching and learning are based on one’s educational philosophy. Scientific constructivism is a philosophy of learning, not a methodology of teaching. Unfortunately, this is misunderstood by many (Clements, n.d.). Learning mathematics using scientific constructivism will focus on helping students construct their own personal meaning of the concepts they are studying. The major tenets of constructivism (Clements & Battista,

1990) include the following: 1) Knowledge creation is an *active* process, not a passive one, 2) Reality is multiple; therefore, mathematics learning consists of the process of adapting to and organizing one's quantitative world, not just discovering preexisting ideas imposed by others (this is the most controversial tenet), 3) Learning is a social process, and 4) A teacher's demands to use prescribed mathematical methods curtail the sense-making activity of the students.

Table 1

A Comparison of Mathematics as Taught in School and as a Discipline

Mathematics in schools	Mathematics as a discipline
Mathematics is neat and concise. It is about memorizing correct procedures or algorithms for solving well-defined problems.	Mathematics is messy. It involves a search for sense and order from complex, ill-defined situations.
Speed and correct answers are emphasized.	Persistence and flexibility are essential to mathematical pursuits. Mathematicians often spend years trying to solve a single problem.
Answers are validated by the teacher or answer book.	There is no answer book. Often there are no "best" answers or even a guarantee that an answer will be found.
Calculators may be used only once basic skills are mastered. Computers and other technologies are useful primarily for drill but also for enrichment.	Tools (manipulatives, computers, calculators) are continuously used to examine and represent ideas or extend thinking. Tedious computations are done by machines. Thinking and reasoning are done by people.
Mathematics is done in isolation, working quietly from a textbook or a worksheet.	Mathematics is a collaborative endeavor with mathematicians and others working together, communicating their ideas and building on one another's ideas and experiences.

Note. From *A New Vision for Mathematics Education in Ohio* (p. 24), by M. T. Battista, 2001. Paper prepared for the Ohio Mathematics and Science Coalition, Kent State University, Ohio, September, 2001. Retrieved from http://www.ohiomsc.org/omsc/PDF/MathVision9-01.pdf

Decades of empirical research (O'Brien, 1999, p. 434) on children's mathematics learning in a constructivist mode have concluded the following: mathematical learning takes place from an interaction between knower and known, children's thinking is very different from adult thinking, and social interaction is a major cause of intellectual growth. These tenets have implications for both student learning and teacher training.

The Student Perspective

What does a constructivist lesson look like? Battista (1999) illustrates the constructivist approach to a problem as follows. Consider the problem, "What is 2 ½ divided by ¼?" Traditionally, students would solve this problem by using the "invert and multiply" method without truly understanding what that means. In contrast, a student who has made sense of fractions will use mental models and will begin by thinking that because there are four fourths in each unit and because there are two fourths in a half, there are 10 fourths in 2 ½ (see Figure 1). This student is purposefully and meaningfully reasoning and making sense of the ideas. Such thinking will "enable the student to apply mathematical knowledge to real-world situations" (Battista, 1999, p. 428). Of course, a student should have knowledge that supports mathematical reasoning, such as, for example, basic number facts.

A powerful means to teach mathematics is by connecting language and cognition to mathematics. Two examples (Hyde, 2007, p. 46) that use reading comprehension strategies to assist in problem solving are as follows:

- **Making Connections**. Teach students to make a variety of connections as they attempt to understand a problem. Representation strategies include discussing in small groups (linguistic/auditory); using manipulatives (concrete/tactile); acting it out (bodily kinesthetic); drawing a picture (visual); or making a list or table (symbolic), each using a different sensory modality.

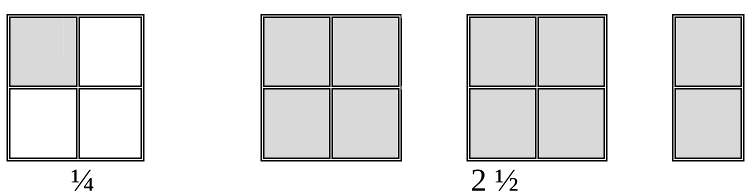

Figure 1. Solving a fraction problem the constructivist way.

- **Inferring and Predicting.** Students can be taught to infer and predict using the modified K-W-L (Know-Want to know-Learned) strategy for reading comprehension which is K-W-C (Know-Want to know—Conditions to watch for). For example, if a problem states, "The car traveled 90 miles in two hours." A student may assert that what we know is the car went 45 miles per hour. Checking whether this is a fact or an inference would elicit a good discussion. That 45 miles/hr is an average (that there could have been a rest stop) would be a possible conclusion, depending on the conditions under which the travel took place.

Two fundamental learning mechanisms of scientific constructivism are abstraction and reflection (Battista, 2001). "Abstraction is the fundamental mental mechanism by which new mathematical knowledge is generated" (p. 429). Different degrees of abstraction range from separating an item from the flow of things to using it in new situations. However, mathematics understanding requires more than abstraction, as Battista (2001) points out. "It requires *reflection*, which is the conscious process of mentally replaying experiences, actions, or mental processes and considering their results or how they are composed. As these acts of reflection are themselves abstraction, they can become the content—what is acted upon—in future acts of reflection and abstraction" (p. 429).

It is heartening to see the mathematics curricula recommended by National Council of Teachers of Mathematics (NCTM) and National Research Council in the United States, which have emphasized mathematical reasoning and problem solving, the basic skills of the 21st century. The standards set by NCTM (as cited in Leinwand & Ginsburg, 2007, p. 33) include both content standards and process standards (see Table 2). The characteristic trait of constructivist mathematics curriculum is its incorporation of process standards along with content standards. These standards are applicable globally and can be adopted in schools. Also valuable to learning mathematics are the five strands of mathematical competency—conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (NRC, as cited in Leinwand & Ginsburg, 2007, p. 33).

An important need in mathematics education in schools is a framework that will integrate these separate lists into a unifying framework. Singapore, considered a world leader in school mathematics education, has developed such an integrated system that has proven to produce results. Widely known as the *Singapore Mathematics*, students in this program have topped the world in mathematical proficiency (Leinwand & Ginsburg, 2007, p. 33). Singapore's mathematical framework is simple and straightforward (see Figure 2) and uses problem solving as the center of the framework. The connection making aspect of this framework makes it worth adopting in any school mathematics program.

Table 2

The National Council of Teachers of Mathematics Standards

Content Standards	Process Standards
Number and operations	Problem solving
Algebra	Reasoning and proof
Geometry	Communication
Measurement	Connections
Data analysis and probability	Representations

Note. From "Learning from Singapore Mathematics" (p. 33) by S. Leinwand & A. L. Ginsburg, 2007, *Educational Leadership*, *65*(3), 32-36.

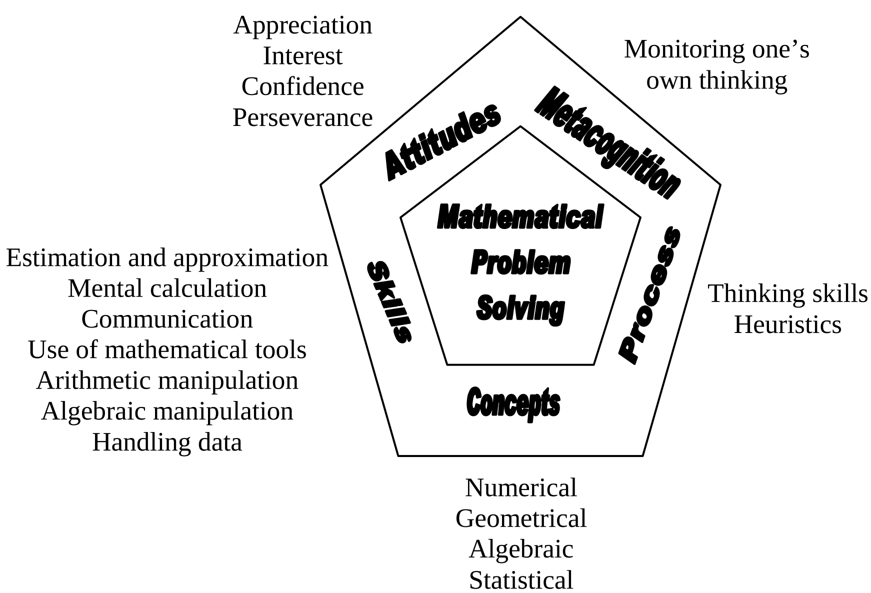

Figure 2. *Singapore mathematics framework.*

Note. From "Learning from Singapore Mathematics" (p. 33) by S. Leinwand & A. L. Ginsburg, 2007, *Educational Leadership*, *65*(3), 32-36.

Besides the use of an organizing framework for mathematics, the success of Singapore Mathematics is also attributed to curriculum alignment, narrow focus in grade levels, multiple models, and rich problems. Leinwand and Ginsburg (2007, p. 34) describe each of these elements as follows: First, curriculum alignment is carefully carried out with each part of the system—the framework, a national set of standards, texts, tests, and teacher preparation programs. Second, the scope of the curriculum in each grade level is less, allowing for deeper meaning making opportunities within the content. Third, the strength of Singapore Mathematics, according to Leinwand and Ginsburg (2007), is the use of a multiple models or representations to explain concepts and to build skills. However, a consistent, single model—the bar, or strip, acts as a unifying pedagogical model to solve problems in “addition, subtraction, multiplication, division, fractions, ratios, and percentages” (p. 35). Fourth, the textbooks in Singapore Mathematics are specifically designed for active thinking processes.

The sequence of content presented in Singapore Mathematics textbooks is concrete to pictorial (colorful) to abstract, which befits the psychological needs of the learners. The textbooks are appraised highly by both mathematicians and teachers (Hoven & Garelick, 2007). These textbooks are also becoming increasingly popular among students because of their simplicity. In reality, the textbooks are replete with complex multistep exercises that develop deeper mathematical understanding (Leinwand & Ginsburg, 2007). Hundreds of schools and homeschoolers in the United States (Hoven & Garelick, 2007), as well as other countries like the Philippines, are currently using these textbooks.

An example (Teach Kids Mathematics with Model Method, n.d.) of a problem using the Singapore Mathematics method is as follows: Paul had 30 marbles. 4/5 of Paul's marbles are equal to 2/3 of John's marbles. How many marbles did John have? Figure 3 shows the use of the bar graph (the pedagogical model) and the multiple steps involved in solving this complex problem. In this problem, students use a multistep procedure, typically using a bar model, to arrive at a solution. A description of the steps might look this: Paul’s 30 marbles when equally divided into 5 parts will result in 30/5 = 6 marbles in each part; so Paul’s 4 parts of marbles will consist of 6 x 4 = 24 marbles=John’s 2/3 part. Since 2 parts for John make 24 marbles, each part for John will be 24/2=12 marbles. Since John has 3 such parts, he has a total of 12 x 3 = 36 marbles. A continuous series of such problems encourages deep understanding in mathematics.

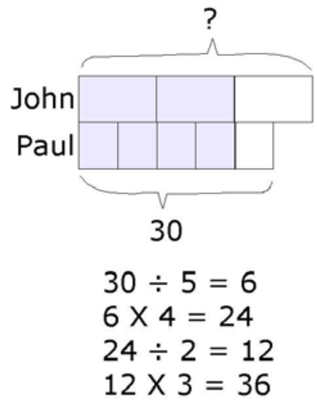

Figure 3. *Solving a complex mathematics problem using the Singapore method.*

Note. From *Teach kids mathematics with model method* (n.d., para. 2). Retrieved from http://www.teach-kids-math-by-model-method.com/fractionsmodel.html

The Teacher's Perspective

The adage "teachers make a difference" is particularly true in mathematics instruction. To be effective, teachers themselves need to have a thorough understanding of the subject matter. Preservice education of teachers should include mathematics content courses and methods courses. Mathematics courses should be taught through inquiry-based methods and must promote the teachers' own personal sense making if there is to be a likelihood for them to do the same for their students. "Teachers themselves need experience in doing mathematics—in exploring, guessing, testing, estimating, arguing, and proving" (NRC, as cited in Lloyd & Frykholm, 2000, para., 2).

Stigler and Hiebert's (2004) report of international comparisons of mathematics achievement pointed out the importance of not only assigning challenging mathematics problems, but also of the need to make connections among mathematics concepts as students solved problems. A research called the Trends in International Mathematics and Science Study (TIMSS) provided a picture of what happens in mathematics classrooms in terms of teachers implementing *making connections* problems in six top mathematics achieving countries (Australia, the Czech Republic, Hong Kong, Japan, the Netherlands, and the United States). It was found that making connections in mathematics problems was actually more important than doing procedural exercises with the problems.

The TIMSS study (Stigler & Hiebert, 2004) reported that in typical classrooms, teachers often change *making connections* problems into problems that use procedures such as using formulas where students are asked to simply use the appropriate values and come up with an answer. For example, in a *making meaning* problem designed to figure out the method for calculating the area of several types of triangles, most teachers may turn the problem into simply calculating the area of the triangle using the formula *½ base x height* and asking students to fill in the values. The findings indicated that the methods of teaching or the ways teachers help students interact with the subject were more important than the kind of curriculum or textbooks used. Building a knowledge base whereby teachers of mathematics can learn how other teachers implement *making connections* problems is also helpful (Stigler & Heibert, 2004).

A problem that still lingers in mathematics education is in the content learning of mathematics teachers. Bass (1997) has observed that most often mathematical preparation of teachers is entrusted to mathematical scientists "who are often neither trained in nor sensitive to the pedagogical aspects of teaching mathematics to young students" (p. 20). While mathematics is an exact science, mathematics education is not. Mathematics education is "more empirical and inherently multidisciplinary," asserts Bass (1997, p. 21). Teaching mathematics content to future teachers of mathematics in schools should include good pedagogical practices and the professional development of mathematics faculty as well as graduate students of mathematics should also include instruction in teaching strategies and communication skills (Bass, 1997).

Several important lessons can be learned from the mathematics education practices of today. The main concerns are the teaching strategies used in schools and the mathematics content learning of future teachers. Mathematics learning can be made effective by using a scientific constructivist approach in both of these areas. Believing that the means to learning mathematics is as important as the end product of learning mathematics, educators can collaboratively create a supportive environment for mathematics learning. Mathematics need not be a subject which is enjoyed and pursued by only a few students. I believe there is hope for Natasha that one day she can exit school mathematically healthy. For that to be possible, all those who are involved in mathematics education need to make changes in the way mathematics is taught—to end the miseducation in mathematics.

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